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Correlations between ε'/ε and Rare K Decays in the Littlest Higgs Model with T-Parity

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Abstract

We calculate the CP-violating ratio ε'/ε in the Littlest Higgs model with T-parity (LHT) and investigate its correlations with the branching ratios for $K_L \to \pi^0\nu\bar{\nu}$, $K_L \to \pi^0\ell^+\ell^-$ and $K^+ \to \pi^+\nu\bar{\nu}$. The resulting correlations are rather strong in the case of K_L decays, but less pronounced in the case of $K^+ \to \pi^+\nu\bar{\nu}$. Unfortunately, they are subject to large hadronic uncertainties present in ε'/ε, whose theoretical prediction in the Standard Model (SM) is reviewed and updated here. With the matrix elements of $\mathcal{Q}_6$ (gluon penguin) and $\mathcal{Q}_8$ (electroweak penguin) evaluated in the large-N limit and $m_s^{\overline{\rm MS}}(2\,{\rm GeV}) = 100\,{\rm MeV}$ from lattice QCD, $(\varepsilon'/\varepsilon)_{\rm SM}$ turns out to be close to the data so that significant departures of $Br(K_L \to \pi^0\nu\bar{\nu})$ and $Br(K_L \to \pi^0\ell^+\ell^-)$ from the SM expectations are unlikely, while $Br(K^+ \to \pi^+\nu\bar{\nu})$ can be enhanced even by a factor 5. On the other hand, modest departures of the relevant hadronic matrix elements from their large-N values allow for a consistent description of ε'/ε within the LHT model accompanied by large enhancements of $Br(K_L \to \pi^0\nu\bar{\nu})$ and $Br(K_L \to \pi^0\ell^+\ell^-)$, but only modest enhancements of $Br(K^+ \to \pi^+\nu\bar{\nu})$.

Note added

An additional contribution to the Z penguin in the Littlest Higgs model with T-parity has been pointed out in [1, 2], which has been overlooked in the present analysis. This contribution leads to the cancellation of the left-over quadratic divergence in the calculation of some rare decay amplitudes. Instead of presenting separate errata to the present work and our papers [3–6] partially affected by this omission, we have presented a corrected and updated analysis of flavour changing neutral current processes in the Littlest Higgs model with T-parity in [7].

1 Introduction

Flavour Changing Neutral Current (FCNC) processes provide a powerful tool for testing the Standard Model (SM) and its extensions. Of particular interest are the four rare kaon decays $K_L \to \pi^0\nu\bar{\nu}$, $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0 e^+e^-$ and $K_L \to \pi^0\mu^+\mu^-$. Their branching ratios are strongly suppressed within the SM and consequently can be largely modified by New Physics (NP) contributions.

Extensive analyses of these decays in the MSSM [8], the Littlest Higgs model with T-parity (LHT) [3], general models with enhanced Z-penguin contributions [9] and Z'-models [10] have shown that in the presence of new sources of flavour and CP-violation beyond those present in the Minimal Flavour Violation (MFV) framework [11, 12], enhancements of $Br(K_L \to \pi^0\nu\bar{\nu})$ by an order of magnitude and of the other branching ratios by up to a factor 5 are still possible.

On the other hand, as pointed out in [13] and analyzed in more detail within the MSSM in [14], the enhancements of the rare decay branching ratios in question could be bounded in principle by the value of ε'/ε that measures the ratio of the direct and indirect CP-violating contributions to $K_L \to \pi\pi$. The reason is very simple. The electroweak penguin and box diagrams that enter the evaluation of the rare decay branching ratios in question have also considerable impact on the ratio ε'/ε so that, in a given model, specific correlations between ε'/ε and the branching ratios for rare K decays exist.

Unfortunately, whereas the branching ratios of $K \to \pi\nu\bar{\nu}$ decays are theoretically very clean [15] and those of $K_L \to \pi^0\ell^+\ell^-$ are subject to only moderate theoretical uncertainties [16], this is not the case for the ratio ε'/ε that is affected by large hadronic uncertainties.

Indeed, whereas the Wilson coefficients of the local operators entering the evaluation of ε'/ε are known [17–22] at the NLO level in QCD and QED renormalization group improved perturbation theory, the hadronic matrix elements of these operators are still only poorly known.[1] Therefore the predictions for ε'/ε in the SM and its extensions have very large theoretical uncertainties.

In spite of this unsatisfactory situation and in view of future improvements in the evaluation of the relevant hadronic matrix elements by lattice QCD or large-N methods, we think that it is important to analyze the correlations between ε'/ε and rare kaon decays in specific extensions of the SM, where large enhancements of the rare decay branching ratios have been found. Certainly, the result of such an exercise will sensitively depend on the values of the hadronic parameters present in ε'/ε, but the mere fact that such correlations exist will hopefully motivate further non-perturbative studies.

The main goal of the present paper is the calculation of ε'/ε within the LHT model

[1] Latest reviews can be found in [23–27].

[28,29] and the investigation of its correlations with the four rare kaon decays in question, for a given set of the non-perturbative parameters entering ε'/ε. To this end we will apply a useful parameterization of ε'/ε proposed in [23] that automatically takes into account all renormalization group effects from scales below m_t and expresses the hadronic uncertainties in terms of the two parameters R_6 and R_8 corresponding to the dominant QCD and electroweak penguin operators, respectively.

In [3] very sharp and theoretically clean correlations between the decays $K_L \to \pi^0\nu\bar{\nu}$, $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\ell^+\ell^-$ have been found in the LHT model, subject mainly to a discrete ambiguity present in the correlation between $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$. It is therefore sufficient to establish the correlations between ε'/ε and $K_L \to \pi^0\nu\bar{\nu}$ and between ε'/ε and $K^+ \to \pi^+\nu\bar{\nu}$ in order to get an idea about all correlations.

Our paper is organized as follows. In Section 2 we briefly review the status of ε'/ε in the SM, investigate the relevant theoretical and parametric uncertainties and provide a numerical update of [23]. In Section 3 we present the basic formulae for ε'/ε in a generic model with new complex phases but no new operators relative to the SM, in terms of the short distance functions X, Y, Z and E that contain both SM and NP contributions. It turns out that in the LHT model the functions X, Y and Z can directly be obtained from our previous analysis [3]. The function E that plays a subdominant role in ε'/ε, is calculated for completeness here for the first time in the LHT model. In Section 4 we evaluate ε'/ε in the LHT model scanning over its parameters and for various values of R_6 and R_8. The main results of this section are the correlations between ε'/ε and $Br(K_L \to \pi^0\nu\bar{\nu})$ and between ε'/ε and $Br(K^+ \to \pi^+\nu\bar{\nu})$, that illustrate the very important role of ε'/ε in bounding the enhancements of rare K decay branching ratios provided the non-perturbative parameters R_6 and R_8 are accurately known. We conclude in Section 5.

2 ε'/ε in the SM

2.1 Basic Formula

Before analyzing ε'/ε within the LHT model, it will be instructive to have a brief look at this ratio within the SM and investigate the relevant theoretical and parametric uncertainties that have to be taken into account also in the case of the LHT model. This will also allow us to update the analysis of [23].

The formula for ε'/ε of [23] is given in the SM as follows:

$$\frac{\varepsilon'}{\varepsilon} = \mathrm{Im}(\lambda_t) \cdot \Big[P_0 + P_E E_0(x_t) + P_X X_0(x_t) + P_Y Y_0(x_t) + P_Z Z_0(x_t) \Big] \tag{2.1}$$

with $\lambda_t = V_{ts}^* V_{td}$ and $x_t = m_t^2/M_W^2$. The short distance physics is described by the loop functions $E_0(x_t)$, $X_0(x_t)$, $Y_0(x_t)$ and $Z_0(x_t)$, for which explicit expressions can be found in [30]. On the other hand, the P_i encode information about the physics at scales $\mu \leq \mathcal{O}(m_t, M_W)$, and are given in terms of the hadronic parameters

$$R_6 \equiv B_6^{(1/2)} \left[\frac{121\,\mathrm{MeV}}{m_s(m_c)+m_d(m_c)}\right]^2, \quad R_8 \equiv B_8^{(3/2)} \left[\frac{121\,\mathrm{MeV}}{m_s(m_c)+m_d(m_c)}\right]^2 \tag{2.2}$$

as follows:

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8. \tag{2.3}$$

The coefficients $r_i^{(0)}$, $r_i^{(6)}$ and $r_i^{(8)}$ enclose information on the Wilson-coefficient functions of the $\Delta S = 1$ weak effective Hamiltonian at the next-to-leading order [30]. Their numerical values for different choices of $\Lambda_{\overline{\mathrm{MS}}}^{(4)}$ at $\mu = m_c$ in the NDR renormalization scheme can be found in [23]. The numerical values of the P_i are sensitive functions of R_6 and R_8, as well as of $\Lambda_{\overline{\mathrm{MS}}}^{(4)}$ or equivalently $\alpha_s(M_Z)$. The values $\Lambda_{\overline{\mathrm{MS}}}^{(4)} = 310, 340, 370\,\mathrm{MeV}$ considered in [23] and by us correspond to the three-loop values $\alpha_s(M_Z) = 0.119, 0.121, 0.123$, respectively. The two-loop formula for the strong coupling constant, instead, provides $\alpha_s(M_Z) = 0.117, 0.119, 0.121$. Although three-loop values are quoted by the PDG [31,32], we use in the present analysis the two-loop values, as the Wilson coefficients entering ε'/ε are known at the NLO only.

2.2 Status of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ from Lattice QCD

The hadronic parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ represent the matrix elements of the dominant QCD penguin operator $\mathcal{Q}_6$ and the dominant EW penguin operator $\mathcal{Q}_8$, respectively. They are the main source of uncertainty in the determination of ε'/ε and, hence, calculating $\langle \pi\pi | \mathcal{Q}_{6,8} | K \rangle$ reliably represents a theoretical challenge for the non-perturbative methods like lattice QCD and large-N. The large-N approach will be referred to below while we focus here on the status of lattice studies relevant for ε'/ε. The lattice calculation of the $\mathcal{Q}_6$ matrix element is particularly delicate. Golterman and Pallante [33], indeed, have pointed out that there is a serious ambiguity in the lattice version of left-right QCD penguin operators, like $\mathcal{Q}_6$, because the flavour group in (partially) quenched QCD is not $SU(3)$ but $SU(3+N_f|3)$ where N_f is the number of sea quark flavours. It turns out that the ambiguity in $\mathcal{Q}_6$ has such a large effect on ε'/ε that it can even flip its sign in quenched QCD [34–36]. Moreover, the same problem affects the left-left QCD penguin operator $\mathcal{Q}_4$ with a sub-leading effect in ε'/ε [37]. On the other hand, the lattice calculation of the $\mathcal{Q}_8$ matrix element is more reliable, although challenging as well and still affected by an uncertainty of $10 \div 20\%$. Two independent approaches have been used. In the indirect approach, one calculates the hadronic matrix elements

of $K \to \pi$ and $K \to 0$ and reconstructs $K \to \pi\pi$ amplitudes using chiral perturbation theory. This method, relatively easy and computationally cheap, has been widely used [34, 38, 39], but it only works in leading order chiral perturbation theory. In the direct approach, instead, one calculates directly the $K \to \pi\pi$ matrix elements with the final state pions carrying a physical momentum. The difficulty of this method is represented by the Maiani-Testa "no-go theorem" [40]: one can not obtain $K \to \pi(\vec{p})\pi(-\vec{p})$ but only $K \to \pi(\vec{0})\pi(\vec{0})$ on the lattice, where $|\pi(\vec{0})\pi(\vec{0})\rangle$ is the ground state of two pions with periodic boundary condition in the spatial direction. Various methods have been proposed to get around the Maiani-Testa theorem. Luscher and Wolff [41] proposed a diagonalization method, based on a computationally expensive calculation of correlators with non-zero pion momentum. Another possibility consists in modifying the boundary condition for the pions [42], thus providing a finite momentum to the ground state of $\pi^{\pm}$. A different approach was elaborated by Lellouch and Luscher [43], based on an excited state fit to extract the $|\pi(\vec{p})\pi(-\vec{p})\rangle$ state that appears in a finite volume where the spectrum of two-particle states is discrete, and on a formula for connecting the decay measured in a finite volume to the infinite volume result, in the center of mass (CM) frame. This technique, however, is challenging due to the need to extract the excited state. An alternative and promising method is to work with a kinematic setup for which the final state of interest is also the lowest energy state. This has been done in [44–46] by working in the moving (LAB) frame, i.e. calculating $\langle\pi(\vec{P})\pi(\vec{0})|\mathcal{Q}_8|K(\vec{P})\rangle$ and then converting the result from the finite to the infinite volume, using the Lellouch-Luscher formula [43]. An important theoretical advance of the last years is the derivation of a relationship similar to the Lellouch-Luscher formula but valid in the LAB frame [47, 48] that may improve the accuracy of the LAB-frame method, as shown in a preliminary calculation with domain wall fermions [49].

2.3 Comparison between SM Prediction and Experimental Data

On the experimental side, the world average based on the latest results from NA48 [50] and KTeV [51] and previous results from NA31 [52] and E731 [53] reads

$$\varepsilon'/\varepsilon = (16.7 \pm 1.6) \cdot 10^{-4}. \tag{2.4}$$

While several analyses made in recent years within the SM found results that are compatible with (2.4), it is fair to say that the large hadronic uncertainties in the coefficients P_i still allow for sizeable NP contributions. The relevant list of references can be found in [23–27].

In [23] an agreement of the SM with (2.4) has been found for $(R_6, R_8) = (1.2, 1.0)$ and $\Lambda^{(4)}_{\overline{\rm MS}} = (340 \pm 30)\,\mathrm{MeV}$. Meanwhile the value of m_t decreased and the value of $\mathrm{Im}(\lambda_t)$

increased. Consequently for $R_6 = R_8 = 1.0$, corresponding to the large-N approach of [54] with $B_6^{(1/2)} = B_8^{(3/2)} = 1$, and $m_s^{\overline{\rm MS}}(2\,{\rm GeV}) = 100\,{\rm MeV}$ from lattice QCD [31, 55],[2] acceptable agreement with (2.4) can be obtained, provided $\Lambda^{(4)}_{\overline{\rm MS}} > 340\,{\rm MeV}$. Indeed in this case we find for $\Lambda^{(4)}_{\overline{\rm MS}} = 340\,{\rm MeV}$

$$P_0 = 15.962\,, \quad P_X = 0.597\,, \quad P_Y = 0.519\,, \quad P_Z = -12.416\,, \quad P_E = -1.226\,, \tag{2.5}$$

and choosing ${\rm Im}(\lambda_t) = 1.38 \cdot 10^{-4}$, obtained by the UTfit collaboration [57], the result

$$(\varepsilon'/\varepsilon)_{\rm SM} = 12.3 \cdot 10^{-4} \tag{2.6}$$

which is a bit lower than the value in (2.4). For $\Lambda^{(4)}_{\overline{\rm MS}} = 370\,{\rm MeV}$ we find, on the other hand,

$$(\varepsilon'/\varepsilon)_{\rm SM} = 13.5 \cdot 10^{-4}\,, \tag{2.7}$$

within 2σ from the central value in (2.4). A slight decrease of the $m_s^{\overline{\rm MS}}(2\,{\rm GeV})$ value (see Table 1) would result in an improved agreement with the data.

We would like to emphasize, then, that with ${\rm Im}(\lambda_t) = 1.69 \cdot 10^{-4}$, obtained from the tree level determination of the CKM parameters, the values in (2.6) and (2.7) increase to

$$(\varepsilon'/\varepsilon)_{\rm SM} = 15.3 \cdot 10^{-4}\,, \tag{2.8}$$

and

$$(\varepsilon'/\varepsilon)_{\rm SM} = 16.7 \cdot 10^{-4}\,, \tag{2.9}$$

so that even for $\Lambda^{(4)}_{\overline{\rm MS}} = 340\,{\rm MeV}$ and $m_s^{\overline{\rm MS}}(2\,{\rm GeV}) = 100\,{\rm MeV}$ a good agreement with the data can be obtained.

As a preparation for the analysis of ε'/ε in the LHT model we show in Fig. 1 the values of $(\varepsilon'/\varepsilon)_{\rm SM}$ for three different choices of $(R_6, R_8) = (1.0, 1.0), (1.5, 0.8), (2.0, 1.0)$, different values of $\Lambda^{(4)}_{\overline{\rm MS}}$ and the two values for ${\rm Im}(\lambda_t)$ considered above.

The main messages from Fig. 1 and (2.6)-(2.9) are:

- $(\varepsilon'/\varepsilon)_{\rm SM}$ has a visible dependence on the values chosen for ${\rm Im}(\lambda_t)$ and for $\Lambda^{(4)}_{\overline{\rm MS}}$, but these dependences amount only to about $10 \div 20\%$, which is comparable to the experimental error in (2.4).

- $(\varepsilon'/\varepsilon)_{\rm SM}$ depends very strongly on the values of R_6 and R_8, and the choices $(1.5, 0.8)$ and $(2.0, 1.0)$ give values for $(\varepsilon'/\varepsilon)_{\rm SM}$ that clearly are in disagreement with the data for the full range of $\Lambda^{(4)}_{\overline{\rm MS}}$ and ${\rm Im}(\lambda_t)$ considered by us. For instance for $\Lambda^{(4)}_{\overline{\rm MS}} = 340\,{\rm MeV}$ and the UTfit value of ${\rm Im}(\lambda_t)$ one finds $(\varepsilon'/\varepsilon)_{\rm SM} = 26.3 \cdot 10^{-4}$ and $(\varepsilon'/\varepsilon)_{\rm SM} = 36.5 \cdot 10^{-4}$ for $(R_6, R_8) = (1.5, 0.8)$ and $(R_6, R_8) = (2.0, 1.0)$, respectively.

[2]Similar results are found from QCD sum rules [56].

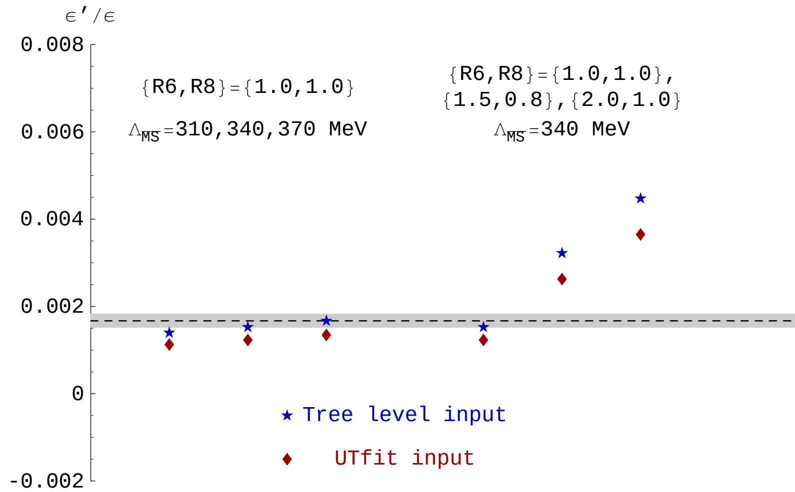

Figure 1: $(\varepsilon'/\varepsilon)_{SM}$ *for three different choices of* $(R_6, R_8) = (1.0, 1.0), (1.5, 0.8), (2.0, 1.0)$ *and different values of* $\Lambda^{(4)}_{\overline{\rm MS}} = 310, 340, 370\,{\rm MeV}$*. The values obtained with the UTfit value for* $\mathrm{Im}(\lambda_t)^{UTfit} = 1.38\cdot 10^{-4}$ *are marked with red diamonds, while those with the tree level value* $\mathrm{Im}(\lambda_t)^{tree} = 1.69\cdot 10^{-4}$ *are marked with blue stars. The shaded area represents the experimental result in* (2.4)*.*

- Significant although smaller departures of (R_6, R_8) from $(1.0, 1.0)$ and therefore of ε'/ε from the data could also occur for $B_6^{(1/2)} = B_8^{(3/2)} = 1$, as obtained from the large-N approach of [54], and values of the strange quark mass deviating from $m_s^{\overline{\rm MS}}(2\,{\rm GeV}) = 100\,{\rm MeV}$ by the present $10 \div 20\%$ lattice uncertainty (see Table 1).

$m_s^{\overline{\rm MS}}(2\,{\rm GeV})$	80 MeV	90 MeV	100 MeV	110 MeV	120 MeV
(R_6, R_8)	$(1.5, 1.5)$	$(1.2, 1.2)$	$(1.0, 1.0)$	$(0.8, 0.8)$	$(0.7, 0.7)$

Table 1: *Choices for the strange quark mass within present lattice uncertainties and corresponding values for the hadronic parameters* (R_6, R_8)*. The small down quark mass has a minor impact and its value is fixed to* $m_d^{\overline{\rm MS}}(2\,{\rm GeV}) = 5\,{\rm MeV}$ *[31]. The variation of* $\Lambda^{(4)}_{\overline{\rm MS}}$ *entering the quark mass running represents a small effect as well and its value is fixed to* $\Lambda^{(4)}_{\overline{\rm MS}} = 340\,{\rm MeV}$*.*

As reviewed in [23], $R_8 = 1.0 \pm 0.2$ is obtained in various approaches. Unfortunately the value of R_6 is very uncertain. For instance in the large-N approach of [58, 59] values for R_6 significantly higher than 1 have been found. In particular [58] reports $R_6 = 2.2 \pm 0.4$ and $R_8 = 1.1 \pm 0.3$. On the other hand, while the lattice values of R_8 are compatible with 1 [38, 44], they are lower than unity for R_6 [34, 35].

3 ε'/ε in the LHT Model

The LHT model [28, 29] belongs to the class of Little Higgs models [60], where the little hierarchy problem is solved by a naturally light Higgs, identified with a Nambu-Goldstone boson of a spontaneously broken global symmetry. In the LHT model the global group $SU(5)$ is spontaneously broken into $SO(5)$ at the scale $f \approx \mathcal{O}(1\,\mathrm{TeV})$ and the electroweak sector of the SM is embedded in an $SU(5)/SO(5)$ non-linear sigma model. Gauge and Yukawa Higgs interactions are introduced by gauging the subgroup of $SU(5)$: $[SU(2)\times U(1)]_1 \times [SU(2)\times U(1)]_2$, such that the so-called collective symmetry breaking prevents the Higgs from becoming massive when the couplings of one of the two gauge factors vanish. A discrete symmetry called T-parity [29] is then introduced, in order to reconcile the model with electroweak precision tests. It restores the custodial $SU(2)$ symmetry and, therefore, the compatibility with electroweak precision data is obtained already for quite small values of the NP scale, $f \geq 500\,\mathrm{GeV}$ [61, 62]. Another important consequence is that particle fields are T-even or T-odd under T-parity. The particles belonging to the T-even sector are the SM particles and a heavy top T_+, while the T-odd sector consists of heavy gauge bosons $W_H^\pm, Z_H, A_H$, a scalar triplet Φ, an odd heavy top T_- and the so-called mirror fermions [63], i.e., fermions corresponding to the SM ones but with opposite T-parity and $\mathcal{O}(1\,\mathrm{TeV})$ mass. Mirror fermions are characterized by new flavour and CP-violating interactions with SM fermions and heavy gauge bosons, thus allowing significant effects in flavour observables [3–5, 64–66] without new operators in addition to the SM ones.

The formula for the CP-violating ratio ε'/ε of [23] in a generic model with new complex phases but no new operators, like the LHT model, generalizes as follows:

$$\frac{\varepsilon'}{\varepsilon} = \frac{\mathrm{Im}(\lambda_t)}{\sin(\beta-\beta_s)}\,\tilde{F}_{\varepsilon'}(v), \tag{3.1}$$

with $\lambda_t = V_{ts}^* V_{td}$, $\beta_s = -1.3^\circ$ and

$$\begin{aligned}\tilde{F}_{\varepsilon'}(v) = {} & P_0 \sin(\beta-\beta_s) + P_E\,|E_K| \sin\beta_E^K \\ & + P_X\,|X_K| \sin\beta_X^K + P_Y\,|Y_K|\sin\beta_Y^K + P_Z\,|Z_K|\sin\beta_Z^K\,,\end{aligned} \tag{3.2}$$

where β is the angle in the unitarity triangle to be specified below (see Table 2).

P_i are the same as in the SM while the short distance physics is now described by the loop functions

$$X_K = |X_K|\,e^{i\,\theta_X^K}\,, \qquad Y_K = |Y_K|\,e^{i\,\theta_Y^K}\,, \qquad Z_K = |Z_K|\,e^{i\,\theta_Z^K}\,, \qquad E_K = |E_K|\,e^{i\,\theta_E^K}\,, \tag{3.3}$$

that are generalizations of the real valued SM loop functions X_0, Y_0, Z_0 and E_0 in (2.1) to the LHT model. Explicit expressions for X_K, Y_K and Z_K have been obtained in [3]. The

function E_K can be found, in complete analogy to the functions $T_{D'}$ and $T_{E'}$ governing the $B \to X_s\gamma$ decay [64], by changing the argument of the SM E_0 function and properly adjusting various overall factors. The result is given in Appendix A. The phases β_i^K entering (3.2) are then given by

$$\beta_i^K = \beta - \beta_s - \theta_i^K \qquad (i = X, Y, Z, E)\,. \tag{3.4}$$

A comment on two approximations made above is in order. The first one concerns the contributions from the T-even sector to the functions X_K and Y_K. In the calculation of these functions, the fermion mass on the flavour conserving side of the box diagrams has been set to zero, since in the case of semileptonic rare decays SM leptons are present. On the other hand, in the case of non-leptonic decays, such as $K_L \to \pi\pi$, this mass cannot be generally neglected, as now up-type quarks, in particular the top quark and the heavy T_+, contribute. However, it can straightforwardly be shown that including this difference results in the presence of a new operator [67]

$$(\bar{s}d)_{V-A}(\bar{b}b)_{V-A} \tag{3.5}$$

at scales $\mu > m_b$, which is not contained in (2.1) and (3.1), (3.2). It is multiplied by the function

$$S_t = S_0(x_t) + \bar{S}_{\rm even}\,, \tag{3.6}$$

where $S_0(x_t)$ denotes the SM contribution and $\bar{S}_{\rm even}$ the heavy T_+ contribution. Below the scale $\mu = m_b$ the b quark is integrated out, and therefore the operator in (3.5) contributes to ε'/ε only through mixing under renormalization. In the case of the SM, this contribution has been shown to be $\mathcal{O}(1\%)$ and therefore fully negligible [67]. As in the LHT model the dominant contribution to S_t comes from the SM part $S_0(x_t)$ [64,68], the accuracy of neglecting this contribution remains the same in the LHT model.

The second approximation entering the above formula (3.2) concerns the T-odd sector and consists in neglecting the mass splittings of mirror quarks on the flavour conserving side of the box diagrams contributing to the X_K and Y_K functions. We have checked, see also [3], that the inclusion of these splittings affects the functions X_K and Y_K by at most 10%. As P_X and P_Y are much smaller than P_0 and $|P_Z|$, these functions do not play a dominant role in ε'/ε anyway and we can safely neglect also this effect in view of large non-perturbative uncertainties.

In the LHT model, the first term in (3.2), which involves P_0 and is dominated by the QCD penguin operator $\mathcal{Q}_6$, does not contain any NP contribution. On the other hand, the important negative last term involving P_Z and related to the EW penguin operator $\mathcal{Q}_8$ can be strongly enhanced, when $\theta_Z \neq 0$, $\sin\beta_Z \simeq 1$ and $|Z| > Z_0(x_t)$. These conditions can indeed be satisfied, as found in [3] from a general scan over the three

generation mirror fermion masses and the six parameters (three angles θ^d_{12}, θ^d_{13}, θ^d_{23} and three phases δ^d_{12}, δ^d_{13}, δ^d_{23}) of the mixing matrix V_{Hd}. Thus, in this case, the suppression of ε'/ε through the enhanced electroweak penguin contribution must be compensated by the increase of the QCD penguin contribution P_0 or by decreasing the magnitude of the coefficient P_Z. This corresponds to the increase of R_6 and the decrease of R_8, respectively.

Clearly, as seen in the previous section, the result for ε'/ε is very sensitive to the actual values of the coefficients P_i. In the LHT model, in addition, there is a strong dependence on the phases β^K_i.

We conclude this section commenting on the origin of the correlations present in the LHT model between ε'/ε and rare kaon decays. They come from the simultaneous dependence of rare K decays and ε'/ε on the short-distance functions X_K, Y_K and Z_K. For instance, the branching ratio for $K_L \to \pi^0\nu\bar{\nu}$ reads

$$Br(K_L \to \pi^0\nu\bar{\nu}) = \kappa_L \tilde{r}^2 A^4 R_t^2 |X_K|^2 \sin^2\beta^K_X \,, \tag{3.7}$$

where [69]

$$\kappa_L = (2.22 \pm 0.07)\cdot 10^{-10}\,, \qquad \tilde{r} = \left|\frac{V_{ts}}{V_{cb}}\right| \simeq 0.98\,, \qquad R_t = \frac{|V_{td}V^*_{tb}|}{|V_{cd}V^*_{cb}|} \simeq 1.0\,. \tag{3.8}$$

As, in the LHT model, there are also strong correlations between X_K, Y_K and Z_K, in particular between their phases, it is evident that there will be a strong correlation between the CP-violating observables ε'/ε and $Br(K_L \to \pi^0\nu\bar{\nu})$.

The explicit expressions for $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_L \to \pi^0\ell^+\ell^-)$ in terms of X_K, Y_K and Z_K are more complicated than the one in (3.7). They are given in [3], to which we refer for details, and forecast as well correlations between ε'/ε and these decays.

4 Numerical Analysis in the LHT Model

In our numerical analysis in the LHT model presented below we have used for the determination of the CKM parameters, and in particular of $\mathrm{Im}(\lambda_t)$, the tree level values of $|V_{ub}|$, $|V_{cb}|$, λ and γ given in Table 2, as the UTfit values obtained within the SM are clearly not valid in the LHT model. In obtaining the SM values of rare decay branching ratios in Table 3 below, however, we consistently used the determination of the CKM parameters within the SM. As a curiosity we remark that with the CKM values of Table 2, due to an increased value of $\mathrm{Im}(\lambda_t)$ with respect to the UTfit determination, the SM branching ratios are higher than those given in Table 3 and their central values

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$$Br(K_L \to \pi^0 \nu\bar{\nu})^{\rm tree}_{\rm SM} = 4.0 \cdot 10^{-11}\,, \qquad Br(K^+ \to \pi^+ \nu\bar{\nu})^{\rm tree}_{\rm SM} = 9.5 \cdot 10^{-11}\,, \tag{4.1}$$

$$Br(K_L \to \pi^0 e^+e^-)^{\rm tree}_{\rm SM} = 3.8 \cdot 10^{-11}\,, \qquad Br(K_L \to \pi^0 \mu^+\mu^-)^{\rm tree}_{\rm SM} = 1.5 \cdot 10^{-11}\,. \tag{4.2}$$

However, such a procedure would not be fully consistent as the CKM values in Table 2 deviate significantly from the SM UTfit values: the reason is the so-called "$\sin 2\beta$ problem" [70].

$G_F = 1.16637 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$		$\Delta M_K = 3.483(6) \cdot 10^{-15}\,\mathrm{GeV}$	
$M_{\rm W} = 80.425(38)\,\mathrm{GeV}$		$\Delta M_d = 0.508(4)/\mathrm{ps}$	[72]
$\alpha = 1/127.9$		$\Delta M_s = 17.77(12)/\mathrm{ps}$	[73, 74]
$\sin^2\theta_W = 0.23120(15)$	[31]	$S_{\psi K_S} = 0.675(26)$	[72]
$\lvert V_{ub}\rvert = 0.00409(25)$		$F_K\sqrt{\hat B_K} = 143(7)\,\mathrm{MeV}$	[31, 75]
$\lvert V_{cb}\rvert = 0.0416(7)$	[72]	$F_{B_d}\sqrt{\hat B_{B_d}} = 214(38)\,\mathrm{MeV}$	
$\lambda = \lvert V_{us}\rvert = 0.2258(14)$		$F_{B_s}\sqrt{\hat B_{B_s}} = 262(35)\,\mathrm{MeV}$	[75]
$\gamma = 82(20)^\circ$	[57]	$\eta_1 = 1.32(32)$	[76]
$m_{K^0} = 497.65(2)\,\mathrm{MeV}$		$\eta_3 = 0.47(5)$	[77]
$m_{D^0} = 1.8645(4)\,\mathrm{GeV}$		$\eta_2 = 0.57(1)$	
$m_{B_d} = 5.2794(5)\,\mathrm{GeV}$		$\eta_B = 0.55(1)$	[78]
$m_{B_s} = 5.370(2)\,\mathrm{GeV}$		$\overline{m}_{\rm c} = 1.30(5)\,\mathrm{GeV}$	
$\lvert\varepsilon_K\rvert = 2.284(14) \cdot 10^{-3}$	[31]	$\overline{m}_{\rm t} = 161.7(20)\,\mathrm{GeV}$	

Table 2: *Values of the experimental and theoretical quantities used as input parameters.*

The discussion of Sections 2 and 3 forecasts that in order to allow large enhancements of the rare decays $K_L \to \pi^0\nu\bar{\nu}$ and $K_L \to \pi^0\ell^+\ell^-$, the consistency with the data on ε'/ε requires $R_6 > R_8$. In Fig. 2 we show ε'/ε as a function of $Br(K_L \to \pi^0\nu\bar{\nu})$ in the LHT model for different values of (R_6, R_8). To this end we have set $\Lambda_{\overline{\rm MS}} = 340\,\mathrm{MeV}$ and performed a general scan over the parameters of the LHT model subject to present experimental constraints from K and B physics as discussed in detail in [3, 64]. We compare the plot resulting from the general scan with the one obtained setting to zero two phases, δ^d_{12} and δ^d_{23}, of the V_{Hd} mixing matrix.[3] These two plots are significantly different, signaling that ε'/ε is quite sensitive to the new phases δ^d_{12} and δ^d_{23}, whereas this sensitivity was much weaker in the case of rare decays discussed in [3]. This shows that ε'/ε is not only very sensitive to the values of the hadronic matrix elements but also to the new parameters of a given model. This fact could be used in the future to

[3]A detailed analysis of the number of phases in the mixing matrices in the LHT model has been presented in [71].

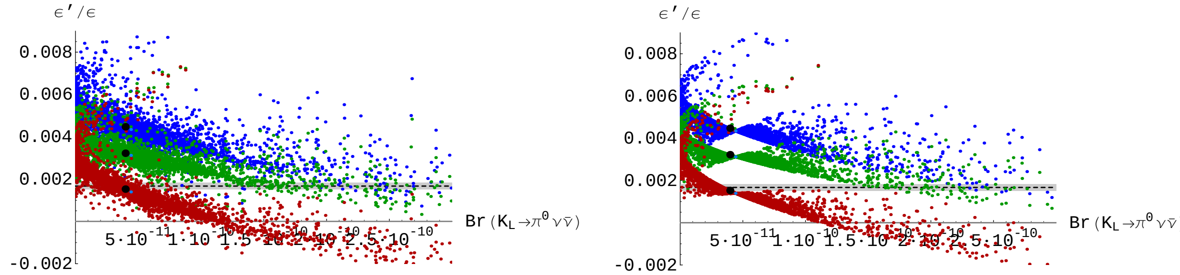

Figure 2: *Left:* ε'/ε *as a function of* $Br(K_L \to \pi^0 \nu\bar{\nu})$ *for different values of* $(R_6, R_8) = (1.0, 1.0)$ *(red),* $(1.5, 0.8)$ *(green),* $(2.0, 1.0)$ *(blue). The shaded area represents the experimental result in* (2.4) *while the SM predictions are displayed by the black points. Right: Same as before, but with two phases* (δ^d_{12} *and* δ^d_{23}) *of the mixing matrix* V_{Hd} *set to zero. Comparing the left and right plots, it is evident that* ε'/ε *turns out to be quite sensitive to these phases.*

efficiently exclude some portions of the parameter space provided the hadronic matrix elements will be brought under control.

We observe that for $(R_6, R_8) = (1.0, 1.0)$ (red points), large enhancements of $Br(K_L \to \pi^0 \nu\bar{\nu})$ over the SM value imply a strong suppression of ε'/ε relative to the data, and consequently in this case large enhancements of $Br(K_L \to \pi^0 \nu\bar{\nu})$ found in the LHT model in [3] are unlikely. The same applies to $Br(K_L \to \pi^0 \ell^+ \ell^-)$. On the other hand, for $(R_6, R_8) = (1.5, 0.8)$ (green points) and $(R_6, R_8) = (2.0, 1.0)$ (blue points) the experimental data for ε'/ε imply in the LHT model a significant enhancement of $Br(K_L \to \pi^0 \nu\bar{\nu})$ with respect to the SM.

As $K_L \to \pi^0 \nu\bar{\nu}$ and $K_L \to \pi^0 \ell^+ \ell^-$ are very strongly correlated with each other [3], also $Br(K_L \to \pi^0 \ell^+ \ell^-)$ are predicted to be enhanced for $(R_6, R_8) = (1.5, 0.8)$ and $(R_6, R_8) = (2.0, 1.0)$. We summarize in Table 3 the three choices for (R_6, R_8) and the corresponding values of rare decay branching ratios that are compatible with the data for ε'/ε.

In Fig. 3 we show the correlation between ε'/ε and $K^+ \to \pi^+ \nu\bar{\nu}$ that is significantly weaker than in the case of ε'/ε and $K_L \to \pi^0 \nu\bar{\nu}$. In particular, we find that in the case $(R_6, R_8) = (1.0, 1.0)$, in which $Br(K_L \to \pi^0 \nu\bar{\nu})$ and $Br(K_L \to \pi^0 \ell^+ \ell^-)$ are required to be SM-like, $Br(K^+ \to \pi^+ \nu\bar{\nu})$ can be largely enhanced relative to its SM value. A different behaviour is observed for the two other choices of (R_6, R_8) considered by us. Here only enhancements of $Br(K^+ \to \pi^+ \nu\bar{\nu})$ by at most a factor 3 are allowed.

In order to understand better the pattern of enhancements of $Br(K_L \to \pi^0 \nu\bar{\nu})$ and $Br(K^+ \to \pi^+ \nu\bar{\nu})$, we show in Fig. 4 the correlation between $Br(K_L \to \pi^0 \nu\bar{\nu})$ and

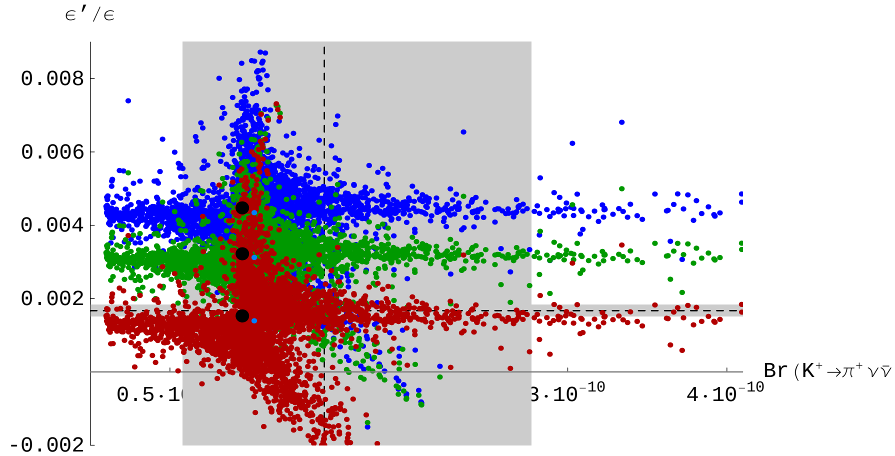

Figure 3: *Correlation between ε'/ε and $K^+ \to \pi^+\nu\bar{\nu}$ for different values of* $(R_6, R_8) = (1.0, 1.0)$ *(red),* $(1.5, 0.8)$ *(green),* $(2.0, 1.0)$ *(blue). The shaded areas represent the experimental results while the SM predictions are displayed by the black points.*

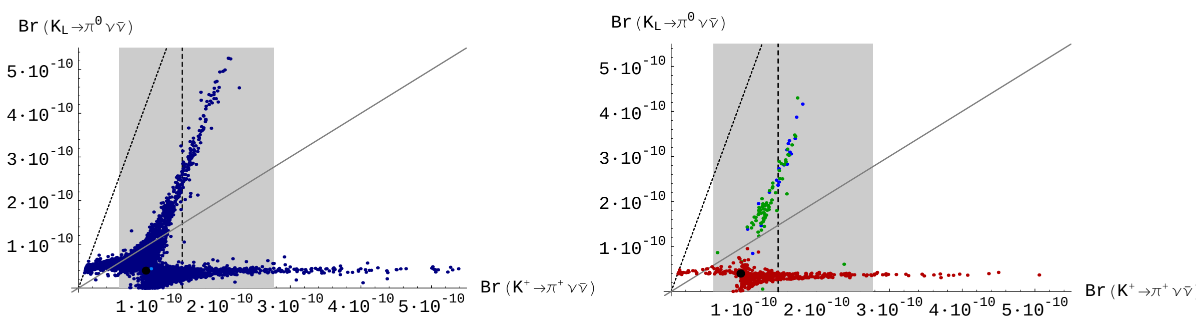

Figure 4: *Left: Correlation between $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ without imposing the ε'/ε-constraint [3]. The shaded area represents the experimental result for $Br(K^+ \to \pi^+\nu\bar{\nu})$ [80] while the SM predictions are displayed by the black points. The Grossman-Nir bound [81] is displayed by the dotted line, while the solid line separates the two areas where $Br(K_L \to \pi^0\nu\bar{\nu})$ is larger or smaller than $Br(K^+ \to \pi^+\nu\bar{\nu})$. Right: Same as before, but after imposing the constraint from ε'/ε with different values of* $(R_6, R_8) = (1.0, 1.0)$ *(red),* $(1.5, 0.8)$ *(green),* $(2.0, 1.0)$ *(blue).*

	SM	$(1.0, 1.0)$	$(1.5, 0.8)$	$(2.0, 1.0)$
$Br(K_L \to \pi^0 \nu\bar{\nu}) \cdot 10^{11}$	2.7 ± 0.4	$0.007 \ldots 9.5$	$0.5 \ldots 43$	$8.4 \ldots 42$
$Br(K^+ \to \pi^+ \nu\bar{\nu}) \cdot 10^{10}$	0.84 ± 0.10	$0.09 \ldots 5.7$	$0.6 \ldots 2.3$	$1.0 \ldots 1.8$
$Br(K_L \to \pi^0 e^+ e^-) \cdot 10^{11}$	$3.54^{+0.62}_{-0.49}$	$2.7 \ldots 4.7$	$2.9 \ldots 8.8$	$4.2 \ldots 8.6$
$Br(K_L \to \pi^0 \mu^+ \mu^-) \cdot 10^{11}$	$1.41^{+0.28}_{-0.26}$	$1.2 \ldots 1.8$	$1.2 \ldots 3.9$	$1.8 \ldots 3.8$

Table 3: *Choices for (R_6, R_8) and the corresponding values of rare decay branching ratios that are compatible with the data for ε'/ε. The SM predictions [79] are also shown. For $Br(K_L \to \pi^0 \ell^+ \ell^-)$ we consider for simplicity only the case of constructive interference between direct and indirect CP-violation.*

$Br(K^+ \to \pi^+ \nu\bar{\nu})$ in the LHT model without the ε'/ε constraint as obtained in [3], and after the constraint from ε'/ε for different choices for (R_6, R_8) has been taken into account. We observe that setting $(R_6, R_8) = (1.0, 1.0)$ basically selects the horizontal branch on which $Br(K_L \to \pi^0 \nu\bar{\nu})$ is SM-like but $Br(K^+ \to \pi^+ \nu\bar{\nu})$ can be strongly enhanced. The other two choices for (R_6, R_8) select the second branch on which $Br(K_L \to \pi^0 \nu\bar{\nu})$ can be strongly enhanced but $Br(K^+ \to \pi^+ \nu\bar{\nu}) < 2.3 \cdot 10^{-10}$.

5 Conclusions

In this paper we have calculated ε'/ε for different values of the hadronic parameters (R_6, R_8) in the LHT model and investigated the implications for rare decay branching ratios when taking the experimental data for ε'/ε into account. The main results of our paper are given in Figs. 2–4 and in Table 3 and can be summarized as follows:

- For the values of hadronic parameters $(R_6, R_8) \simeq (1.0, 1.0)$, for which $(\varepsilon'/\varepsilon)_{\rm SM}$ agrees with the data, large enhancements of $Br(K_L \to \pi^0 \nu\bar{\nu})$ and $Br(K_L \to \pi^0 \ell^+ \ell^-)$ relative to the SM are very unlikely.

- On the other hand, for the values of hadronic parameters $(R_6, R_8) = (1.5, 0.8)$ and $(2.0, 1.0)$ chosen by us, the large NP contributions that are required to fit the experimental value for ε'/ε result in large enhancements of $Br(K_L \to \pi^0 \nu\bar{\nu})$ and $Br(K_L \to \pi^0 \ell^+ \ell^-)$ relative to the SM.

- The correlation between ε'/ε and $K^+ \to \pi^+ \nu\bar{\nu}$ is much weaker and large departures of $Br(K^+ \to \pi^+ \nu\bar{\nu})$ from the SM values are possible even for $(R_6, R_8) \simeq (1.0, 1.0)$, however, more modest enhancements are possible for the other choices of hadronic parameters, as seen in Figs. 3 and 4.

The main message of our paper is clear: without significant progress in the evaluation of R_6 and R_8 and other less important hadronic parameters entering ε'/ε, the role of the data in (2.4) in constraining physics beyond the SM will remain limited.

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A Explicit Formulae for the Function E_K

In this appendix we give the explicit expression for the function E_K entering the calculation of ε'/ε in the LHT model. The functions X_K, Y_K and Z_K have been calculated already in [3] in the context of rare K and B decays and can be found in that paper. The variables are defined as follows:

$$x_t = \frac{m_t^2}{M_{W_L}^2}\,, \qquad x_T = \frac{m_{T_+}^2}{M_{W_L}^2}\,, \tag{A.1}$$

$$z_i = \frac{m_{Hi}^2}{M_{W_H}^2}\,, \qquad z_i' = a\, z_i \quad \text{with } a = \frac{5}{\tan^2\theta_W} \qquad (i=1,2,3)\,, \tag{A.2}$$

$$\lambda_t = V_{ts}^* V_{td}\,, \qquad \xi_i^{(K)} = V_{Hd}^{*is} V_{Hd}^{id} \qquad (i=1,2,3)\,, \tag{A.3}$$

and x_L describes the mixing in the T-even top sector.

$$E_0(x_t) = -\frac{2}{3}\log x_t + \frac{x_t^2(15-16x_t+4x_t^2)}{6(1-x_t)^4}\log x_t + \frac{x_t(18-11x_t-x_t^2)}{12(1-x_t)^3}\,, \tag{A.4}$$

$$E_K = E_0(x_t) + \bar{E}_{\text{even}} + \frac{1}{\lambda_t}\bar{E}_K^{\text{odd}}\,, \tag{A.5}$$

$$\bar{E}_{\text{even}} = x_L^2 \frac{v^2}{f^2}\left[E_0(x_T) - E_0(x_t)\right]\,, \tag{A.6}$$

$$\bar{E}_K^{\text{odd}} = \frac{1}{4}\frac{v^2}{f^2}\sum_i \xi_i^{(K)}\left[\frac{3}{2}E_0(z_i) + \frac{1}{10}E_0(z_i')\right]\,. \tag{A.7}$$

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